Available on: https://jsshrs.ep-journals.org/ E-ISSN: 3051-3561, P-ISSN: 3051-3553

Journal of Social Science and Human Research Studies

Volume : 02 Issue : 09 September-2026

Exploring the Integration of Language Literacy and Numeracy in Junior High School Students' Conceptual Understanding of Geometry Based on Pirie-Kieren Theory

Muchtadi
Department of Magister Mathematics Education, Faculty of Mathematics, Natural Sciences and Technology, Universitas PGRI Pontianak, Indonesia

Dewi Risalah*
Department of Magister Mathematics Education, Faculty of Mathematics, Natural Sciences and Technology, Universitas PGRI Pontianak, Indonesia

Hariyadi
Department of Indonesian Language and Literature Education, Universitas PGRI Pontianak, Indonesia

Syarifah Fadillah Alhadad
Department of Magister Mathematics Education, Faculty of Mathematics, Natural Sciences and Technology, Universitas PGRI Pontianak, Indonesia

Safuan
Master Program of Mathematics Education, Universitas PGRI Pontianak, Indonesia

Published Date: September 07, 2026 **Article DOI: 10.65150/EP-jsshrs/V2E9/2026-05**

ABSTRACT : This article presents an exploratory qualitative study designed to examine how language literacy and numeracy are integrated in junior high school students' conceptual understanding of geometry through the lens of Pirie-Kieren theory. The study is motivated by persistent difficulties among students in understanding geometric concepts, connecting mathematical representations, and communicating mathematical ideas in a logical and systematic manner. In geometry learning, students are required not only to perform numerical procedures but also to read mathematical texts, interpret symbols, coordinate visual representations, and construct meaning from verbal, symbolic, and spatial information. The study uses a descriptive-exploratory qualitative design involving seventh-grade students at SMPN 21 Pontianak. At the initial stage, 60 students are involved in a conceptual understanding test and classroom observation on the topic of surface area of flat-sided solid figures. Six students are then selected purposively for in-depth analysis based on variations in their conceptual understanding. Data are collected through conceptual understanding tests, literacy-based tasks, classroom observation, semi-structured interviews, and students' written artifacts. Data analysis is conducted through data reduction, open coding, axial coding, selective coding, interpretation, and triangulation based on the eight layers of Pirie-Kieren mathematical understanding. The expected contribution of this study is a conceptual model explaining the interaction between language literacy, numeracy, and students' geometric understanding, including the identification of conceptual and linguistic obstacles that emerge during geometry learning.

KEYWORDS: language literacy, numeracy, conceptual understanding, geometry, Pirie-Kieren theory, junior high school

INTRODUCTION

Conceptual understanding is a fundamental competency in mathematics education because it allows students to understand the meaning of concepts, explain relationships among mathematical ideas, and apply knowledge flexibly in new situations. In geometry, conceptual understanding becomes particularly important because geometric tasks require students to coordinate spatial reasoning, visualization, representation, measurement, and mathematical communication. However, many junior high school students still experience difficulties in learning geometry. Their difficulties are not limited to computational errors; they also include misunderstanding the meaning of geometric concepts, failing to connect verbal, visual, and symbolic representations, and being unable to justify solutions logically.

The problem indicates that geometry learning should not be viewed merely as the mastery of formulas. When students learn the surface area of flat-sided solid figures, for example, they need to interpret verbal information, recognize two-dimensional nets, imagine three-dimensional structures, identify relevant properties, and connect these representations to numerical procedures. Therefore, students' mathematical understanding is closely related to their ability to read, interpret, and communicate mathematical language. In this sense, language literacy is not an external element of mathematics learning; rather, it is part of how mathematical meaning is constructed.

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

Numeracy also plays a central role in this process. Numeracy enables students to use quantitative relationships, interpret numerical information, and make decisions based on mathematical reasoning. In geometry learning, numeracy supports students in understanding dimensions, calculating areas, selecting formulas appropriately, and evaluating the reasonableness of their answers. Nevertheless, numeracy alone is insufficient when students cannot understand the language of the problem or explain the conceptual basis of their procedures. The integration of language literacy and numeracy is therefore essential to support deeper and more meaningful geometry learning.

One theoretical framework that can explain the development of mathematical understanding is Pirie-Kieren theory. This theory describes mathematical understanding as a dynamic, layered, and non-linear process consisting of eight layers: primitive knowing, image making, image having, property noticing, formalising, observing, structuring, and inventising. A distinctive feature of this theory is folding back, a process in which students return to earlier layers of understanding to reconstruct, refine, or deepen their understanding. This perspective is relevant to geometry learning because students often move back and forth among visual images, properties, symbols, and formal reasoning.

Previous studies have investigated mathematical literacy, numeracy, conceptual understanding, and the Pirie-Kieren theory separately. However, research that explicitly integrates language literacy and numeracy within the Pirie-Kieren framework to analyze junior high school students' conceptual understanding of geometry remains limited. This gap is important because conceptual difficulties in geometry may arise not only from cognitive or numerical limitations but also from linguistic barriers, such as misunderstanding task statements, misinterpreting mathematical terms, or failing to express reasoning clearly.

Based on this background, this study aims to explore the integration of language literacy and numeracy in junior high school students' conceptual understanding of geometry based on Pirie-Kieren theory. Specifically, the study seeks to analyze the development of students' geometric conceptual understanding, examine the role of language literacy in the construction of mathematical understanding, identify the relationship between language literacy and mathematical representation, and reveal conceptual and linguistic obstacles experienced by students in geometry learning.

LITERATURE REVIEW

Conceptual Understanding in Geometry

Conceptual understanding refers to students' ability to grasp the meaning of mathematical concepts, recognize relationships among concepts, and represent those concepts in multiple forms. It differs from procedural knowledge because it emphasizes why a mathematical procedure works, how ideas are connected, and how a concept can be transferred to new contexts. Hiebert and Grouws (2007) argue that conceptual understanding involves the construction of connected mathematical ideas that form a coherent knowledge structure.

In geometry, conceptual understanding includes the ability to interpret spatial forms, recognize properties of shapes, connect two-dimensional and three-dimensional representations, and reason about measurements. Students with strong conceptual understanding are able to explain why a formula is appropriate, identify relationships among parts of a solid figure, and justify their solutions. Conversely, students with weak conceptual understanding tend to memorize formulas without understanding the geometric meaning behind them, which may lead to errors when the context of a problem changes.

Pirie-Kieren Theory of Mathematical Understanding

Pirie-Kieren theory provides a useful framework for analyzing the growth of mathematical understanding. The theory views understanding as an organic, recursive, and dynamic process that develops through eight layers. The first layer, primitive knowing, refers to the initial knowledge students bring to a learning situation. Image making occurs when students construct mental or physical images through activities. Image having refers to students' ability to use these images without always relying on concrete actions. Property noticing emerges when students identify characteristics or relationships among mathematical objects. Formalising involves the use of mathematical language, rules, and symbols. Observing occurs when students reflect on and coordinate formalized ideas. Structuring refers to the organization of mathematical ideas into a coherent system. Finally, inventising represents the creation of new mathematical insights or generalizations.

A central concept in Pirie-Kieren theory is folding back. Folding back does not represent failure; instead, it is a productive process in which students return to earlier layers to reconstruct understanding. For example, when students cannot explain a surface area formula symbolically, they may need to return to visual nets or physical models to rebuild the meaning of the formula. In this study, folding back is treated as an important indicator of how students use language, representation, and numeracy to deepen their geometry understanding.

Language Literacy in Mathematics Learning

Language literacy in mathematics learning refers to students' ability to read, understand, interpret, and communicate mathematical information effectively. Mathematical tasks are often presented through written texts, diagrams, tables, symbols, and contextual situations. Therefore, students need to understand not only ordinary language but also the specialized language of mathematics. Terms such as surface area, edge, face, net, base, height, and dimension require precise interpretation in geometry learning.

Language literacy supports students in identifying what is known, what is asked, and what relationships are relevant in a mathematical problem. It also helps students explain their reasoning, justify their procedures, and communicate mathematical ideas to others. When language literacy is weak, students may misread the problem, misunderstand key terms, or fail to connect verbal information with visual and symbolic representations. Such difficulties can hinder conceptual understanding even when students possess basic computational skills.

Numeracy and Representation in Geometry

Numeracy is the ability to use mathematical concepts, procedures, facts, and tools to solve problems in various contexts. In geometry, numeracy involves understanding measurement, operating with formulas, interpreting quantitative relationships, and evaluating solutions. Numeracy is closely related to mathematical representation because students must translate contextual information into diagrams, symbols, and calculations.

The integration of numeracy and representation is essential in learning the surface area of flat-sided solid figures. Students need to interpret dimensions, identify the relevant faces of a solid, determine the area of each face, and combine the results appropriately. This process requires coordination among verbal, visual, symbolic, and numerical representations. Consequently, numeracy should be developed together with language literacy so that students can understand both the quantitative and linguistic dimensions of geometry tasks.

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

Integration of Language Literacy and Numeracy in the Pirie-Kieren Framework

Integrating language literacy and numeracy within the Pirie-Kieren framework allows researchers to analyze students' understanding more holistically. Language literacy helps students interpret mathematical situations and express conceptual meaning, while numeracy helps them operate with quantitative relationships and evaluate mathematical solutions. The Pirie-Kieren layers provide a structure for identifying how these two forms of literacy function at different levels of understanding.

For instance, at the image making layer, language literacy may appear in students' ability to describe geometric objects, while numeracy may appear in their ability to identify dimensions. At the property noticing layer, language literacy may support students in naming and explaining properties, while numeracy supports the recognition of relationships among measurements. At the formalising layer, students are expected to connect verbal explanations, symbols, and formulas. This integrated perspective makes it possible to identify whether students' obstacles are primarily conceptual, linguistic, numerical, representational, or a combination of these.

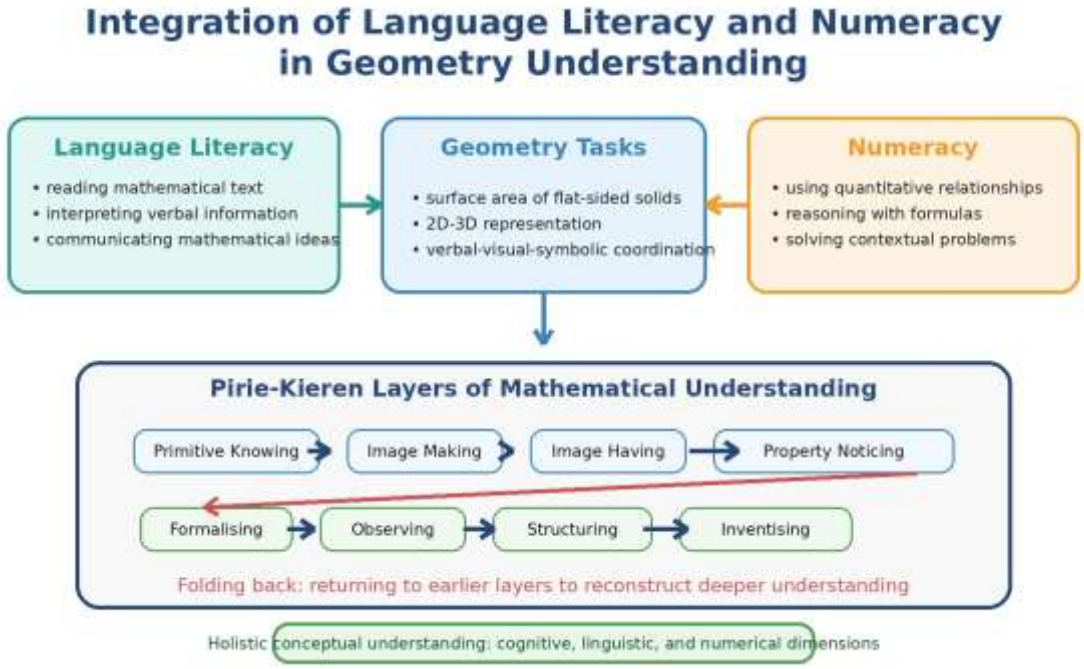


Figure 1. Conceptual framework for integrating language literacy and numeracy in Pirie-Kieren based geometry understanding

MATERIALS AND METHODS

Research Design

This study employs a qualitative approach with a descriptive-exploratory design. The qualitative approach is considered appropriate because the study aims to investigate students' thinking processes, language use, representational coordination, and conceptual obstacles in a natural learning context. The descriptive-exploratory design is used to provide a comprehensive description of how students construct geometric understanding and how language literacy and numeracy contribute to this process.

The study is not intended to test a statistical hypothesis. Instead, it focuses on interpreting students' learning processes, identifying patterns of understanding, and explaining the interaction between cognitive and linguistic dimensions in geometry learning. Pirie-Kieren theory serves as the main analytical lens for tracing students' movement across layers of mathematical understanding.

Research Setting and Participants

The research is conducted at SMPN 21 Pontianak, particularly in Grade VII, where students begin to encounter formal geometry learning. The school context is relevant because it has implemented the Merdeka Curriculum, which emphasizes meaningful learning, student-centered instruction, and the development of literacy and numeracy competencies. The initial participants consist of 60 students who study the topic of surface area of flat-sided solid figures. After the initial analysis, six students are selected purposively as the main subjects for in-depth investigation. The selection considers variation in students' levels of conceptual understanding to obtain diverse trajectories of mathematical understanding.

Object of the Study

The object of the study includes four main aspects: students' conceptual understanding of geometry, students' language literacy in understanding and communicating mathematical ideas, students' numeracy in solving contextual geometry problems, and students' conceptual and linguistic obstacles during geometry learning. These objects are analyzed as interconnected dimensions rather than as separate variables.

Data Collection Techniques and Instruments

Data are collected through several techniques. First, a conceptual understanding test is used to identify students' ability to explain concepts, connect representations, and solve geometry problems. The test consists of open-ended tasks so that students' reasoning processes can be examined. Second, literacy-based tasks are used to assess how students read, interpret, and transform mathematical information from verbal statements into visual and symbolic representations.

Third, classroom observation is conducted to record students' activities, interactions, representational use, and responses during geometry learning. Fourth, semi-structured interviews are conducted with the six selected students to explore their reasoning, language use, folding back processes,

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

and obstacles more deeply. Fifth, students' written artifacts, such as worksheets, test answers, drawings, and notes, are analyzed as supporting documents.

Tabel 1. Alignment of Research Questions, Data Sources, and Analytical Focus

Research Question	Data Source	Pirie-Kieren Focus	Literacy-Numeracy Focus
How does students' conceptual understanding of geometry develop?	Conceptual test, observation, interview, artifacts	Movement across the eight layers and folding back	Coordination of verbal, visual, symbolic, and numerical information
What is the role of language literacy in constructing mathematical understanding?	Literacy-based tasks and interview	Language use at each layer, especially image making, property noticing, and formalising	Reading, interpreting, explaining, and justifying mathematical ideas
How is language literacy related to mathematical representation?	Written work, drawings, interview transcript	Transitions between image having, property noticing, and formalising	Translation among verbal, visual, symbolic, and numerical representations
What conceptual and linguistic obstacles are experienced by students?	Observation, test answers, interview, artifacts	Breakdowns and folding back points	Misinterpretation of terms, representation errors, formula misuse, and communication barriers

Data Analysis

Data analysis is conducted in two stages. In the first stage, the conceptual understanding test results from 60 students are analyzed descriptively to map initial levels of understanding and to select six students for further investigation. In the second stage, verbal, written, and visual data from the selected students are analyzed in depth using open coding, axial coding, and selective coding.

Open coding is used to identify units of meaning related to students' conceptual understanding, language use, numeracy, representation, and obstacles. Axial coding connects the initial codes to the eight layers of Pirie-Kieren theory: primitive knowing, image making, image having, property noticing, formalising, observing, structuring, and inventising. Selective coding is used to integrate the categories into major themes that explain students' trajectories of understanding. Special attention is given to folding back processes, especially when students return to earlier representations or explanations to rebuild conceptual meaning.

Trustworthiness of Data

The trustworthiness of the qualitative data is maintained through credibility, dependability, confirmability, and transferability. Credibility is strengthened through triangulation of techniques and sources by comparing data from tests, interviews, observations, and students' artifacts. Member checking is conducted by confirming selected interview interpretations with students. Dependability and confirmability are supported through systematic documentation of research procedures, coding decisions, and analytical discussions among the research team. Transferability is addressed by providing a detailed description of the learning context, participant characteristics, task design, and students' thinking trajectories.

RESULTS

Because this manuscript is developed from a research proposal and field data collection has not yet been completed, the results section is presented as an analytical framework and projected research output. The section does not report empirical findings; instead, it describes the expected forms of data interpretation that will be produced after the implementation of the study.

Projected Mapping of Students' Understanding Across Pirie-Kieren Layers

The first projected output is a map of students' conceptual understanding across the eight Pirie-Kieren layers. At the primitive knowing layer, students are expected to rely on prior knowledge about plane figures, area, and basic measurement. At the image making layer, students are expected to construct visual images of solid figures, nets, and faces through learning activities. At the image having layer, students may begin to use mental images of solids without direct manipulation.

At the property noticing layer, students are expected to identify relevant properties, such as the number of faces, congruent faces, parallel edges, and relationships between dimensions. At the formalising layer, students are expected to connect these properties to mathematical formulas for surface area. At the observing and structuring layers, students are expected to evaluate the relationship among formulas, representations, and solutions. The inventising layer may appear when students produce alternative strategies, generalizations, or new explanations beyond the given procedure.

Tabel 2. Coding Framework for Integrated Analysis

Pirie-Kieren Layer	Indicator of Geometry Understanding	Language Literacy Indicator	Numeracy Indicator
Primitive Knowing	Recognizes familiar shapes and prior area concepts	Uses everyday language to describe objects	Identifies basic numbers and measurements
Image Making	Builds images through drawings, nets, or models	Reads and restates task information	Identifies dimensions needed for calculation
Image Having	Uses mental images of solid figures	Explains geometric objects verbally	Matches measurements with corresponding parts

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

Property Noticing	Identifies properties of faces, edges, and relationships	Names and explains properties accurately	Connects properties with quantitative relationships
Formalising	Uses formulas and mathematical symbols meaningfully	Translates verbal information into symbolic form	Applies surface area procedures accurately
Observing	Reflects on procedures and solutions	Justifies reasoning in coherent statements	Checks numerical reasonableness
Structuring	Organizes concepts into a coherent system	Constructs complete mathematical explanations	Compares strategies and generalizes relationships
Inventising	Creates new strategies or generalizations	Communicates original mathematical insights	Extends numerical reasoning to new contexts

Projected Identification of Folding Back Processes

The study is expected to identify folding back as an important mechanism in students' geometry learning. Folding back may occur when students are unable to use a formula meaningfully and need to return to drawings or nets; when students misinterpret a word problem and need to reread the task; or when students produce an incorrect calculation and need to revisit the properties of the solid figure. In this framework, folding back is interpreted as a dynamic interaction among cognitive reconstruction, language reinterpretation, and numerical correction.

Projected Conceptual and Linguistic Obstacles

The study is also expected to reveal two main categories of obstacles. Conceptual obstacles may include students' inability to distinguish surface area from volume, difficulty identifying all faces of a solid figure, or confusion in connecting a net to the original solid. Linguistic obstacles may include misunderstanding key geometric terms, failing to interpret contextual information, or being unable to explain why a formula is used.

These obstacles are likely to appear simultaneously. For example, a student may choose an incorrect formula not only because of weak numeracy but also because the student misreads the phrase total surface area or does not understand the relationship between the text and the visual representation. Therefore, the integrated framework allows obstacles to be analyzed as cognitive-linguistic-numerical phenomena.

DISCUSSION

The proposed integration of language literacy and numeracy within the Pirie-Kieren framework offers a more comprehensive way to analyze students' conceptual understanding of geometry. Geometry learning involves a complex relationship among spatial visualization, language interpretation, representation, and numerical reasoning. By using Pirie-Kieren theory, researchers can trace how students move from informal knowledge to formal mathematical understanding and how they return to earlier layers when they experience obstacles.

The inclusion of language literacy expands the interpretation of mathematical understanding. In many mathematics classrooms, students' errors are often categorized as procedural or computational. However, this study emphasizes that an incorrect solution may emerge from difficulties in reading mathematical texts, interpreting terms, connecting sentences with diagrams, or explaining reasoning. Thus, teachers need to pay attention not only to whether students can calculate correctly but also to whether they can understand and communicate the meaning of the problem.

The inclusion of numeracy strengthens the analysis because geometry tasks require students to use quantitative relationships meaningfully. Students must select relevant measurements, determine the relationship among dimensions, operate with formulas, and evaluate answers. Numeracy becomes more meaningful when it is connected to visual and verbal representations. Therefore, mathematics instruction should encourage students to explain the meaning of each calculation rather than simply apply formulas mechanically.

The framework also has implications for differentiated and deep learning practices. Teachers can design geometry learning activities that begin with concrete models and visual representations, continue with guided reading of mathematical tasks, and move toward symbolic generalization. Students who experience difficulty should be allowed to fold back to earlier representations, such as drawings, physical models, or verbal explanations. This process can help students reconstruct understanding instead of merely correcting answers.

For research methodology, the framework supports a rich qualitative analysis because it combines written responses, oral explanations, observations, and artifacts. This allows researchers to identify not only the final answer but also the pathway by which students construct or fail to construct conceptual understanding. Such analysis is valuable for developing pedagogical recommendations that are responsive to students' cognitive, linguistic, and numerical needs.

CONCLUSION

This article proposes an exploratory qualitative study on the integration of language literacy and numeracy in junior high school students' conceptual understanding of geometry based on Pirie-Kieren theory. The study is grounded in the view that students' difficulties in geometry are not merely procedural but also conceptual, representational, linguistic, and numerical. The integration of language literacy and numeracy provides a broader perspective for understanding how students read mathematical information, construct visual and symbolic representations, use formulas, and communicate reasoning.

The Pirie-Kieren framework enables the analysis of students' understanding as a dynamic process that develops through layered growth and folding back. Through this framework, the study is expected to produce a model that explains how language literacy and numeracy interact in the construction of geometric understanding, particularly in the topic of surface area of flat-sided solid figures. The study is also expected to identify conceptual and linguistic obstacles that can inform more effective geometry instruction.

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

Corresponding Author: Dewi Risalah

RECOMMENDATIONS

Teachers are encouraged to integrate reading, interpretation, representation, and numerical reasoning in geometry learning. Geometry tasks should not only ask students to calculate but also require them to explain concepts, interpret terms, draw representations, and justify solutions.

Learning activities should provide opportunities for students to move among verbal, visual, symbolic, and numerical representations. Students who encounter difficulties should be guided to fold back to earlier forms of understanding, such as drawings, physical models, or verbal explanations.

Future research should implement the proposed framework with complete empirical data and compare students' trajectories of understanding across different levels of conceptual achievement. The findings can be used to develop literacy-numeracy based geometry learning designs for junior high school students.

REFERENCES

1) Battista, M. T. (2017). Geometry and geometric reasoning. In J. Cai (Ed.), Compendium for research in mathematics education (pp. 584-613). National Council of Teachers of Mathematics.
2) Creswell, J. W. (2012). Research design: Qualitative, quantitative, and mixed methods approaches (4th ed.). SAGE Publications.
3) Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2012). How to design and evaluate research in education (8th ed.). McGraw-Hill.
4) Gulkilik, H., Moyer-Packenham, P. S., Ugurlu, H. H., & Yuruk, N. (2020). Examining students' mathematical understanding through PirieKieren theory. International Journal of Mathematical Education in Science and Technology, 51(7), 1031-1048. https://doi.org/10.1080/0020739X.2019.1672901
5) Harel, G. (2008). What is mathematics? A pedagogical answer to a philosophical question. In B. Gold & R. A. Simons (Eds.), Current issues in the philosophy of mathematics from the perspective of mathematicians. Mathematical Association of America.
6) Hatch, J. A. (2002). Doing qualitative research in education settings. State University of New York Press.
7) Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester Jr. (Ed.), Second handbook of research on mathematics teaching and learning (pp. 371-404). Information Age Publishing.
8) Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi. (2022). Kurikulum Merdeka: Panduan pembelajaran dan asesmen. Kemendikbudristek.
9) Kieren, T., & Pirie, S. (2019). Growth of mathematical understanding: How can we characterize it and how can we represent it? Journal of Mathematical Behavior, 55, 100687.
10) Marshall, C., & Rossman, G. B. (2016). Designing qualitative research (6th ed.). SAGE Publications.
11) Moschkovich, J. N. (2015). Academic literacy in mathematics for English learners. Journal of Mathematical Behavior, 40, 43-62.
12) Nabilah, W., Faradiba, S. S., Fuat, F., Risalah, D., & Widjajanti, K. (2024). Profile of students' mathematical literacy based on level of mathematical connection ability. Journal Mathematics of Education, 9(1), 89-107. https://doi.org/10.31327/jme.v9i1.2020 13) National Council of Teachers of Mathematics. (2020). Catalyzing change in middle school mathematics. NCTM.
13) OECD. (2019). PISA 2018 results (Volume I): What students know and can do. OECD Publishing.
14) Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterize it and how can we represent it? Educational Studies in Mathematics, 26(2-3), 165-190. https://doi.org/10.1007/BF01273662
15) Prediger, S., & Wessel, L. (2018). Braiding language and mathematics education: Conceptual framework and examples. Journal of Mathematics Education, 11(1), 1-21.
16) Sfard, A. (2018). Thinking as communicating: Human development, the growth of discourses, and mathematizing. Cambridge University Press.
17) Stacey, K. (2015). The international assessment of mathematical literacy: PISA 2012 framework. Journal of Research in Mathematics Education, 46(5), 523-536.
18) Suryadi, D. (2019). Landasan filosofis penelitian desain didaktis (DDR). Pusat Pengembangan DDR Indonesia. 20) Wijaya, A. (2017). Context-based mathematics tasks in Indonesia. Springer.

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

Corresponding Author: Dewi Risalah