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Dimensional Errors in Corrugated Shipping Carton Design and Their Impact on Container Void Space: A Tolerance Stack-Up and Load-Planning Simulation Analysis

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ABSTRACT:

Purpose: Container cube utilization is commonly treated as a load-planning problem, yet the number of cartons that fit into a container is fixed much earlier, when the shipping carton is dimensioned. This study quantifies how dimensional errors introduced during corrugated carton design and conversion propagate into container void space and freight cost. A deterministic and stochastic model was built for floor-loaded, homogeneous cargo in ISO Series 1 containers, in which outer carton dimensions were linked to board caliper, converting tolerance, panel bulge and specification rounding. Tolerance stack-up along each container axis was evaluated with the root-sum-square (RSS) method at 95% reliability, and loading patterns were generated by a layer-based, two-block guillotine heuristic with a This-Side-Up constraint. Five representative regular slotted container (RSC) designs were analysed through sensitivity analysis, five error scenarios and a 5,000-run Monte Carlo simulation. The results show that load quantity responds to dimensional error as a step function rather than proportionally: an error of 1–4 mm per dimension removed 12.5–30.0% of cartons per container in tightly dimensioned designs. Omitting the caliper of a BC-flute double-wall board reduced load quantity by 25.2–38.7%, while the combined effect of converting tolerance, panel bulge and imperial rounding reduced it by 15.8–32.0%, raising the void ratio from 3.2–13.5% to 27.2–37.6%. For an annual flow of 100,000 cartons, this corresponded to 18–73 additional containers per SKU, and loading-pattern re-optimisation recovered no more than 4.2% of the nominal load quantity. These findings indicate that container void space is largely a design-stage defect, and the paper therefore proposes a permissible-dimension rule that integrates packaging engineering with logistics planning.

KEYWORDS: corrugated carton design, dimensional tolerance, tolerance stack-up, container loading, cube utilization, void space, packaging logistics, freight cost

INTRODUCTION

Containerised shipping moves the large majority of manufactured goods in international trade, and the economics of full-container-load (FCL) shipments rest on a simple ratio: the freight charged per container divided by the number of saleable units inside it. Because ocean carriers price an FCL booking per box rather than per cubic metre, every cubic metre of unused space is paid for but generates no revenue for the shipper. The standardisation of the container since the 1950s has made this ratio highly transparent (Levinson, 2016), and the annual reviews of the United Nations Conference on Trade and Development show that freight rates remain volatile and sensitive to disruption, which magnifies the cost of inefficient loading (UNCTAD, 2024). For export-oriented manufacturing economies such as Vietnam, where footwear, apparel, furniture, electronics and processed food are shipped predominantly as floor-loaded cartons, container utilisation is therefore a direct determinant of landed cost and competitiveness.

In logistics practice, low container utilisation is usually attributed to load planning: poor stowage patterns, mixed shipments, weight limits, or the need for load securing. A large body of operations research has accordingly been devoted to the container loading problem (CLP), and modern heuristics achieve volume utilisation above 90% for many benchmark instances (Bortfeldt & Wäscher, 2013; Zhao et al., 2016). However, these algorithms take the box dimensions as given. When the cargo consists of a single stock-keeping unit (SKU) packed in identical cartons, the maximum achievable load quantity is determined almost entirely by the carton's outer dimensions relative to the inside dimensions of the container. In that case the decisive decision is not made by the load planner at the port but by the packaging designer weeks or months earlier, when the inside dimensions of the regular slotted container (RSC) are specified to the corrugated board converter.

Carton dimensioning is a chain of approximations. Designers start from product or inner-pack dimensions, add internal clearance, choose a board grade and flute profile, and transmit a specification that the converter interprets as inside, outside or manufacturing dimensions. Board caliper, scoring allowances, converting tolerances, panel bulge after filling, moisture-induced swelling and the rounding of metric values into imperial specifications all shift the real outer dimension away from the value used in container load calculations (Twede et al., 2015). Individually these deviations are only a few millimetres. Collectively, and when accumulated across 20 or 30 cartons placed end to end along a 12-metre container, they can exceed the slack available in a tightly designed loading pattern, forcing the loader to drop an entire row, column or layer. The result is a container that is shipped with a substantial volume of empty space even though the load plan looked nearly perfect on paper.

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Despite its practical relevance, this mechanism has received limited systematic attention. Packaging logistics research emphasises the interaction between packaging and logistics activities (Hellström & Saghir, 2007; García-Arca et al., 2014; Pålsson, 2018), while tolerance analysis is a mature discipline in mechanical assembly design (Chase & Parkinson, 1991). The two strands have rarely been combined to explain container void space. The present study addresses this gap and is guided by three research questions: (RQ1) How sensitive is the number of cartons per container to dimensional errors introduced at the carton design and converting stage? (RQ2) Which sources of dimensional error contribute most to container void space, and how do they interact? (RQ3) What design rules can prevent design-induced void space before cartons are produced?

To answer these questions, the paper develops a tolerance stack-up model linked to a container load-planning heuristic and applies it to five representative carton designs through deterministic sensitivity analysis, scenario analysis and Monte Carlo simulation. The contribution is threefold. First, it shows that the relationship between dimensional error and load quantity is discontinuous, which explains why small, apparently acceptable errors produce disproportionately large losses. Second, it decomposes the loss into identifiable error sources that can be assigned to specific actors in the packaging supply chain. Third, it derives a permissible-dimension rule that translates a target loading pattern into maximum allowable carton dimensions, giving designers and logistics managers a common, verifiable specification.

LITERATURE REVIEW

Container loading problem and cube utilisation

Cutting and packing problems were classified systematically by Dyckhoff (1990), and the typology was later refined by Wäscher et al. (2007), who place the single-container loading of identical boxes within the class of identical item packing problems. Early constructive heuristics, notably the wall-building approach of George and Robinson (1980), fill the container in vertical layers across its width and remain the basis of many commercial load-planning tools. Subsequent work introduced metaheuristics such as genetic algorithms (Bortfeldt & Gehring, 2001) and addressed practical constraints including orientation, stability, weight distribution and load bearing (Bischoff & Ratcliff, 1995; Bortfeldt & Wäscher, 2013; Ramos et al., 2016). Comparative reviews report that for weakly heterogeneous cargo the best algorithms approach the theoretical volume bound (Zhao et al., 2016). An important implication, rarely stated explicitly, is that for homogeneous cargo the algorithmic improvement potential is small: once carton dimensions are fixed, the attainable number of cartons is bounded by integer division of the container's inside dimensions, and only mixed orientation patterns can recover part of the remainder (Dowsland & Dowsland, 1992).

Cube utilisation, defined as the ratio of cargo volume to container volume, is the standard key performance indicator in this literature. The complementary measure, void space or dead space, represents capacity that is paid for but unused. In practice void space has secondary consequences beyond freight cost: unfilled gaps must be secured with dunnage or void fill to prevent cargo shift, and loose stowage increases the risk of transit damage (Pålsson, 2018).

Packaging design, packaging logistics and dimensional standards

Packaging logistics treats packaging as a system that must simultaneously protect the product, enable efficient handling and transport, and communicate information (Hellström & Saghir, 2007). Azzi et al. (2012) proposed a general framework in which logistics requirements, including space efficiency, are explicit inputs to packaging design. García-Arca et al. (2014) argued that redesigning packaging for logistics efficiency is one of the most cost-effective sustainability levers in supply chains, while Pålsson (2018) documented that packaging decisions affect transport, warehousing and handling costs far beyond the cost of the packaging material itself. Dimensional modularity is a recurring theme: ISO 3394 defines a modular system of plan dimensions for rigid rectangular transport packages based on 600 × 400 mm, and ISO 668 specifies the external dimensions and ratings of Series 1 freight containers (ISO, 2012; ISO, 2020). However, inside container dimensions are not standardised to the millimetre and vary between manufacturers, so designers usually work from carrier-published values.

On the packaging engineering side, the structural design of corrugated fibreboard cartons is governed by box style codes such as the FEFCO 0201 regular slotted container (FEFCO & ESBO, 2007) and by strength models that relate box compression strength to board properties and box perimeter (McKee et al., 1963; Frank, 2014). This literature recognises that outer dimensions depend on flute profile and board caliper and that converting processes introduce tolerances (Twede et al., 2015). It also recognises that cartons bulge under load and humidity, which reduces stacking strength. Yet the spatial consequences of these deviations at the level of a full container are rarely quantified.

Tolerance analysis

Tolerance stack-up analysis, developed for mechanical assemblies, estimates how individual component tolerances accumulate in an assembly dimension (Chase & Parkinson, 1991). The worst-case method adds tolerances linearly and is conservative; the statistical root-sum-square (RSS) method assumes independent, approximately normal deviations and scales accumulated variation with the square root of the number of components. Statistical process control provides the link between process capability and the expected distribution of manufactured dimensions (Montgomery, 2019). A row of cartons placed end to end in a container is, in effect, a one-dimensional assembly whose length must not exceed the container's inside length, which makes tolerance analysis directly applicable.

Research gap

The review reveals three gaps. First, container loading research assumes exact box dimensions and seldom examines how dimensional uncertainty affects attainable load quantity. Second, packaging logistics research recognises the importance of space efficiency but provides few quantitative tools that connect specific design errors with container-level outcomes. Third, tolerance analysis has not been applied systematically to the carton-in-container system. This study bridges these strands by integrating a tolerance stack-up model with a load-planning heuristic and by evaluating the economic consequences for freight cost.

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MATERIALS AND METHODS

Research design

The study follows a quantitative modelling and simulation design. Because the phenomenon of interest is a deterministic geometric mechanism with stochastic inputs, a computational experiment allows the effect of each error source to be isolated while holding all other factors constant, which is difficult to achieve with field observations. The analysis comprises four experiments: (E1) a deterministic sensitivity analysis of load quantity with respect to a uniform dimensional error; (E2) a scenario analysis of five error sources; (E3) a Monte Carlo simulation of random design errors; and (E4) the derivation of permissible carton dimensions for robust loading. All computations were implemented in Python 3 with NumPy; the random seed was fixed to ensure reproducibility.

Container and loading assumptions

Three ISO Series 1 container types were considered, using typical inside dimensions published by carriers (Table 1). Inside dimensions vary slightly by manufacturer; the values used are representative rather than normative. An operational clearance was deducted from each axis to reflect practical loading: 30 mm in length for door closing, 20 mm in width for loader hand space and wall corrugation, and 50 mm in height for handling the top layer. Cargo was assumed to be floor-loaded, homogeneous and volume-limited rather than weight-limited, which is typical for light consumer goods. Cartons were subject to a This-Side-Up constraint, so the height axis of the carton was always vertical and only rotations in the horizontal plane were allowed.

Table 1. Container types and dimensions used in the model

Container type	Inside L × W × H (mm)	Clearance L/W/H (mm)	Usable L × W × H (mm)	Inside volume (m^3)
20' General Purpose (20'GP)	5,898 × 2,352 × 2,393	30 / 20 / 50	5,868 × 2,332 × 2,343	33.2
40' General Purpose (40'GP)	12,032 × 2,352 × 2,393	30 / 20 / 50	12,002 × 2,332 × 2,343	67.7
40' High Cube (40'HC)	12,032 × 2,352 × 2,698	30 / 20 / 50	12,002 × 2,332 × 2,648	76.4

Source: typical carrier-published inside dimensions; external dimensions per ISO 668:2020.

Carton dimension model

The cartons are RSCs (FEFCO 0201). For an RSC the outer dimensions follow from the inside dimensions and the combined board caliper t: the length and width panels add one board thickness on each side, while the height adds two layers of flaps at the top and two at the bottom. The outer dimensions are therefore modelled as

$$L_o = L_i + 2t + \varepsilon_L, \quad W_o = W_i + 2t + \varepsilon_W, \quad H_o = H_i + 4t + \varepsilon_H \tag{1}$$

where L_i, W_i and H_i are inside dimensions and ε denotes the aggregate deviation from all other sources. Four sources of ε were distinguished. (i) Specification error: the designer uses inside dimensions in the load calculation, or the converter interprets the specification differently, so that the caliper term is omitted. For a BC-flute double-wall board a caliper of t = 7 mm was assumed, giving an omission error of +14, +14 and +28 mm in length, width and height. (ii) Converting tolerance: random deviation of each carton from its nominal dimension caused by slotting, scoring, folding and gluing, modelled as a normal distribution with standard deviation σ = 1.5 mm per dimension, corresponding to a ±4.5 mm (±3σ) process range. (iii) Panel bulge: outward deflection of the length and width panels after filling, stacking or exposure to humidity, modelled as a systematic increase of 6 mm in each horizontal dimension. (iv) Specification rounding: conversion of metric dimensions into an imperial specification rounded up to the nearest half inch, which is common when the buyer issues packaging specifications in inches.

Tolerance stack-up along the container axes

A row of n cartons along a container axis j behaves as a one-dimensional assembly. With independent random deviations, the RSS method gives the accumulated row length at reliability level $1 - \alpha$ as $n \cdot d_j + z_\alpha \cdot \sqrt{n} \cdot \sigma_j$, where d_j is the nominal (or systematically shifted) carton dimension along that axis. The number of cartons that fit along axis j is therefore

$$n_j = \max \{ n \in \mathbb{N} : n \cdot d_j + z_\alpha \cdot \sqrt{n} \cdot \sigma_j \le D_j - c_j \} \tag{2}$$

where D_j is the inside container dimension, c_j the operational clearance and $z_\alpha = 1.645$ for 95% one-sided reliability. Setting $\sigma_j = 0$ yields the deterministic case.

Loading pattern heuristic

Loading patterns were generated by a layer-based heuristic in the spirit of George and Robinson (1980). The number of layers is n_H from Equation (2) applied to the carton height. Within a layer, two pure orientations were evaluated (carton length along the container length, or carton width along the container length), followed by a two-block guillotine pattern in which the first k rows along the container length use orientation A and the remaining length is filled with orientation B. The load quantity is

$$N = n_H \cdot \max_k \left[k \cdot n_W^A + n_L^B (D_L - c_L - k \cdot d_L^A) \cdot n_W^B \right] \tag{3}$$

which includes the pure patterns as special cases (k = 0 or k equal to the maximum number of rows in orientation A). The heuristic was deliberately kept simple and transparent; for homogeneous cartons with a fixed vertical axis it reproduces the patterns used in practice by loading supervisors and most load-planning software.

Performance indicators

Container performance was measured by the load quantity N, the relative loss of load quantity compared with the nominal design, the cube utilisation and the void ratio:

$$\Delta = 1 - N / N_0, \quad U = N \cdot l \cdot w \cdot h / (L_c \cdot W_c \cdot H_c), \quad V = 1 - U \tag{4}$$

where N_0 is the nominal load quantity, l × w × h are the nominal outer dimensions of the carton (the designed payload envelope) and $L_c \times W_c \times H_c$ are the inside dimensions of the container. Using nominal rather than actual carton volume ensures that U reflects productive space: air created by

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an oversized carton is counted as void. The freight cost per carton is $C_u = R / N$, where R is the freight rate per container. To isolate the effect of dimensional error, illustrative rates of USD 1,600 per 20'GP, USD 2,500 per 40'GP and USD 2,800 per 40'HC were used. These are model parameters, not market quotations, and all cost results scale linearly with R.

Case designs

Five carton designs representing major export product categories were constructed (Table 2). Each was dimensioned to be container-optimised at its nominal size, i.e. its outer dimensions were chosen to produce a high-utilisation loading pattern, as a competent designer would do. The designs differ in how much slack remains along each axis, which allows the effect of design tightness to be examined. They are stylised cases rather than observations from particular firms.

Table 2. Case carton designs and nominal loading performance

Case	Product category	Container	Nominal outer L × W × H (mm)	Pattern (L × W × layers)	Slack L/W/H (mm)	N_0 (cartons)	U (%)
A	Footwear master carton	40'HC	600 × 388 × 378	20 × 6 × 7	2 / 4 / 2	840	96.8
B	Apparel carton	40'HC	580 × 385 × 330	31 × 4 × 8 (+ mixed)	67 / 12 / 8	992	95.7
C	Consumer electronics shipper	40'GP	470 × 330 × 290	25 × 7 × 8 (+ mixed)	252 / 22 / 23	1,408	93.5
D	Flat-pack furniture carton	40'HC	1,190 × 575 × 160	10 × 4 × 16	102 / 32 / 88	640	91.8
E	Packaged food carton	20'GP	390 × 290 × 235	15 × 8 × 9	18 / 12 / 228	1,080	86.5

Note: slack is the usable container length, width or height left after the pure pattern; U is relative to the container's inside volume.

Experimental design

Experiment E1 added a uniform error e, from −5 mm to +30 mm in 1 mm steps, to all three outer dimensions and recorded N. Experiment E2 evaluated six scenarios: S0 nominal; S1 caliper omitted (BC flute, t = 7 mm); S2 converting tolerance (σ = 1.5 mm, RSS at 95%); S3 panel bulge (+6 mm in length and width); S4 imperial rounding (up to the next half inch); and S5 the combination of S2, S3 and S4, representing a design in which the caliper was correctly included but the remaining sources were not budgeted. For S0–S5 both the pure-orientation and the mixed-orientation patterns were computed to estimate how much loss can be recovered at the load-planning stage. Experiment E3 drew 5,000 random design errors for each case, with each dimension's systematic error drawn from $N(3, 4^2)$ mm to reflect a slight upward bias typical of added clearance and imprecise specification, combined with the converting tolerance of S2. Experiment E4 inverted Equation (2) to derive the maximum permissible nominal dimension that preserves the nominal loading pattern:

$$d_j^{max} = [\, D_j - c_j - z_\alpha \cdot \sqrt{(n_j)} \cdot \sigma_j \,] / n_j - b_j \quad (5)$$

where n_j is the number of cartons along axis j in the target pattern and b_j is the bulge allowance (6 mm for length and width, zero for height).

RESULTS

Sensitivity of load quantity to dimensional error (E1)

Figure 1 shows the load quantity of each case, relative to its nominal value, as a function of the uniform dimensional error. In all cases the response is a step function. Load quantity remains unchanged as long as the error is absorbed by the slack on every axis and then falls abruptly when the slack on any axis is exhausted. For Case A, whose slack is only 2–4 mm per axis, an error of +1 mm reduced the load from 840 to 594 cartons (−29.3%), and +2 mm reduced it to 588 cartons (−30.0%). Case B lost 12.5% at +2 mm and 28.7% at +4 mm. Case C lost 13.0% at +3 mm and 25.4% at +4 mm. Case E lost 14.2% at +2 mm. Case D, with the largest slack relative to its dimensions, tolerated errors up to +5 mm without loss but then dropped by 6.2% at +6 mm and by 29.7% at +9 mm. Beyond the first step, additional error produced comparatively small further losses until the next threshold was reached. Conversely, a reduction of 1–5 mm in Cases B, C, D and E increased the load quantity by 1.6–13.0%, showing that several nominal designs lay immediately next to a threshold.

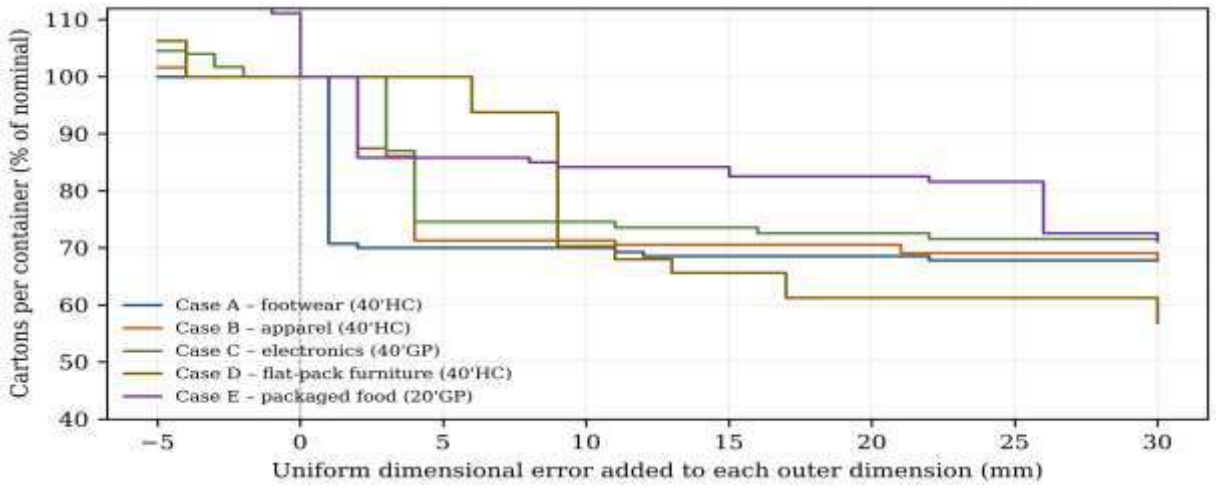

Figure 1. Cartons per container as a function of uniform dimensional error (percentage of nominal load quantity)

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Contribution of individual error sources (E2)

Table 3 reports the load quantity, loss, cube utilisation, void ratio and freight cost per carton for each scenario, and Figure 2 compares the losses across cases. Omitting the board caliper (S1) was the most damaging single error, reducing load quantity by 25.2% to 38.7% and raising the void ratio to between 31.2% and 43.8%. Case D was the most affected because the caliper error exhausted the slack on all three axes at once: the 16-layer stack grew by 448 mm against a height slack of 88 mm, and one carton was lost along both the container length and width.

Converting tolerance alone (S2) affected only Case A, where it reduced load quantity by 29.3%. For the other cases the RSS accumulation of ±1.5 mm deviations over 4 to 31 cartons per row (4.9 to 13.7 mm at 95% reliability) was absorbed by the slack. Panel bulge (S3) caused losses of 14.2% to 18.5% in Cases A, B, C and E but none in Case D. Imperial rounding (S4) had highly case-specific effects: 2.3% for Case C, where the rounded dimensions happened to remain below thresholds, but 25.0% to 30.0% for Cases A and D. The combined scenario (S5) reduced load quantity by 15.8% to 32.0% and increased the void ratio from 3.2–13.5% at nominal to 27.2–37.6%.

The losses were not additive. In Case A, S2, S3 and S4 each produced losses of 18.3% to 30.0%, yet their combination produced 31.4%, almost the same as the largest single source, because once an axis loses a row or layer, further error along the same axis has no additional effect until the next threshold. Conversely, in Case D, S2, S3 and S4 individually produced losses of 0%, 0% and 25.0%, but combined they produced 32.0%, because the bulge consumed the slack that had allowed the rounded dimensions to fit.

Table 3. Load quantity, cube utilisation and freight cost per carton under error scenarios (mixed-orientation pattern)

Case	Scenario	N (cartons)	Loss Δ (%)	U (%)	Void V (%)	Freight per carton (USD)	Recovered by mixed pattern (cartons)
A	S0 Nominal	840	0.0	96.8	3.2	3.33	0
A	S1 Caliper omitted	576	31.4	66.4	33.6	4.86	6
A	S2 Converting tolerance	594	29.3	68.5	31.5	4.71	24
A	S3 Panel bulge	686	18.3	79.1	20.9	4.08	21
A	S4 Imperial rounding	588	30.0	67.8	32.2	4.76	18
A	S5 Combined	576	31.4	66.4	33.6	4.86	6
B	S0 Nominal	992	0.0	95.7	4.3	2.82	0
B	S1 Caliper omitted	700	29.4	67.6	32.4	4.00	0
B	S3 Panel bulge	808	18.5	78.0	22.0	3.47	8
B	S4 Imperial rounding	808	18.5	78.0	22.0	3.47	8
B	S5 Combined	700	29.4	67.6	32.4	4.00	0
C	S0 Nominal	1,408	0.0	93.5	6.5	1.78	8
C	S1 Caliper omitted	1,036	26.4	68.8	31.2	2.41	28
C	S3 Panel bulge	1,200	14.8	79.7	20.3	2.08	0
C	S4 Imperial rounding	1,376	2.3	91.4	8.6	1.82	32
C	S5 Combined	1,022	27.4	67.9	32.1	2.45	14
D	S0 Nominal	640	0.0	91.8	8.2	4.38	0
D	S1 Caliper omitted	392	38.7	56.2	43.8	7.14	14
D	S4 Imperial rounding	480	25.0	68.8	31.2	5.83	0
D	S5 Combined	435	32.0	62.4	37.6	6.44	30
E	S0 Nominal	1,080	0.0	86.5	13.5	1.48	0
E	S1 Caliper omitted	808	25.2	64.7	35.3	1.98	24
E	S3 Panel bulge	927	14.2	74.2	25.8	1.73	45
E	S4 Imperial rounding	927	14.2	74.2	25.8	1.73	27
E	S5 Combined	909	15.8	72.8	27.2	1.76	27

Note: scenarios with zero loss (S2 for Cases B–E; S3 for Case D) are omitted for brevity. Freight per carton uses the illustrative rates stated in the Methods. The last column gives the cartons gained by the two-block pattern over the best pure-orientation pattern.

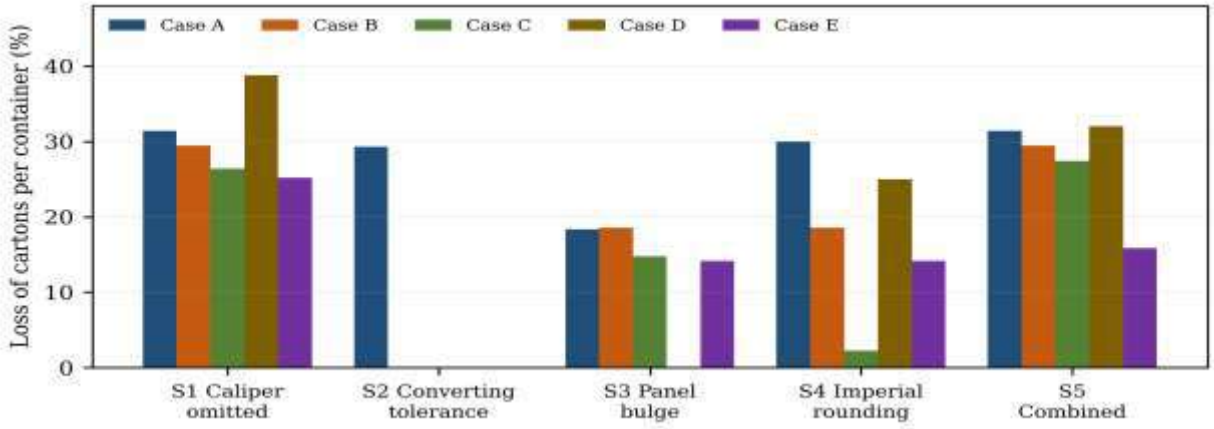

Figure 2. Loss of cartons per container by error scenario and case

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Recovery potential of load-planning optimisation

The last column of Table 3 shows how many cartons the two-block mixed-orientation pattern added compared with the best pure pattern. The gains ranged from 0 to 45 cartons per container, equivalent to no more than 4.2% of the nominal load quantity (Case A, S2: 24 cartons or 2.9%; Case E, S3: 45 cartons or 4.2%). In 8 of the 20 error scenarios shown, the mixed pattern recovered nothing or fewer than 10 cartons. Load-planning optimisation therefore mitigates, but cannot reverse, design-induced void space.

Monte Carlo simulation of random design errors (E3)

Table 4 summarises the distribution of outcomes over 5,000 random designs per case. Even though the mean systematic error was only +3 mm, the probability of losing at least one carton relative to the nominal design ranged from 48.2% (Case D) to 99.4% (Case A). The probability of a loss of at least 10% ranged from 19.3% (Case D) to 96.1% (Case A). The mean loss of load quantity was 6.6% to 24.5%, and the 95th percentile loss was 15.8% to 30.0%. The distributions were strongly bimodal: outcomes clustered either at the nominal quantity or at the next lower step, with few intermediate values, consistent with the step behaviour observed in E1.

Table 4. Monte Carlo results for random design errors (5,000 runs per case)

Case	N_0	Mean N	Minimum N	P(loss > 0) (%)	P(loss ≥ 5%) (%)	P(loss ≥ 10%) (%)	Mean loss (%)	95th percentile loss (%)
A	840	634	576	99.4	96.2	96.1	24.5	30.0
B	992	829	700	96.3	84.7	84.7	16.4	28.7
C	1,408	1,196	1,022	92.6	82.0	81.8	15.1	25.4
D	640	598	420	48.2	48.2	19.3	6.6	29.7
E	1,080	987	900	86.9	60.8	57.2	8.6	15.8

Note: systematic error per dimension ~ N(3, 4^2) mm; converting tolerance σ = 1.5 mm, RSS at 95% reliability; seed = 2026.

Permissible carton dimensions (E4)

Table 5 reports the maximum permissible nominal outer dimension for each axis that preserves the nominal loading pattern under converting tolerance (σ = 1.5 mm, 95% reliability) and a 6 mm bulge allowance. A negative margin indicates that the nominal design is not robust. Cases A, B and E had negative margins of 4.2 to 6.5 mm on both horizontal axes, meaning that their nominal patterns can be achieved reliably only if the product or inner pack is reduced, or if bulge is suppressed through board grade or internal fitments. Case C had a negative margin on the width axis only (−3.8 mm). Case D had positive margins on all axes, although its width margin was only 0.8 mm, which explains its lower but still substantial probability of loss in E3.

Table 5. Maximum permissible outer dimensions for robust loading (mm)

Case	Target pattern (along L × along W × layers)	Nominal dims (along L / along W / H)	Permissible dims (along L / along W / H)	Margin (along L / along W / H)
A	20 × 6 × 7	600 / 388 / 378	593.5 / 381.7 / 377.4	−6.5 / −6.3 / −0.6
B	31 × 4 × 8	385 / 580 / 330	380.7 / 575.8 / 330.1	−4.3 / −4.2 / +0.1
C	25 × 7 × 8	470 / 330 / 290	473.6 / 326.2 / 292.0	+3.6 / −3.8 / +2.0
D	10 × 4 × 16	1,190 / 575 / 160	1,193.4 / 575.8 / 164.9	+3.4 / +0.8 / +4.9
E	15 × 8 × 9	390 / 290 / 235	384.6 / 284.6 / 259.5	−5.4 / −5.4 / +24.5

Note: permissible dimensions from Equation (5) with z = 1.645, σ = 1.5 mm and bulge allowance b = 6 mm (length, width) or 0 mm (height).

Shipment-level economic impact

To express the results at the level of a supply chain, an annual flow of 100,000 cartons per SKU was assumed. Table 6 shows the number of containers required and the additional freight cost caused by dimensional errors. Scenario S5 increased the number of containers by 18 (Case E) to 73 (Case D), or by 19.4% to 46.5%, and the additional freight cost at the illustrative rates ranged from USD 28,800 to USD 204,400 per SKU per year. Scenario S1 increased container requirements by 33.3% to 63.1%.

Table 6. Containers and additional freight cost for an annual flow of 100,000 cartons per SKU

Case	Container	Containers at S0	Containers at S1	Containers at S5	Increase S5 (%)	Additional freight S1 (USD)	Additional freight S5 (USD)
A	40'HC	120	174	174	45.0	151,200	151,200
B	40'HC	101	143	143	41.6	117,600	117,600
C	40'GP	72	97	98	36.1	62,500	65,000
D	40'HC	157	256	230	46.5	277,200	204,400
E	20'GP	93	124	111	19.4	49,600	28,800

Note: illustrative freight rates of USD 1,600 (20'GP), 2,500 (40'GP) and 2,800 (40'HC) per container; results scale linearly with the rate.

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DISCUSSION

Why small errors cause large void space

The central finding is that load quantity is a discontinuous function of carton dimensions. Because the number of cartons along each container axis is an integer, the container behaves like a set of three gauges: any error that exhausts the slack on one axis removes a whole row, column or layer, which for a 20 × 6 × 7 pattern is 5%, 17% or 14% of the load respectively, and errors that are common to all dimensions frequently exhaust the slack on two or three axes at once. This explains the apparently paradoxical observation in practice that cartons which are only a few millimetres larger than specified can leave a container one-quarter to one-third empty. The result also qualifies the common view in the container loading literature that high utilisation is mainly a matter of algorithmic quality (Bortfeldt & Wäscher, 2013; Zhao et al., 2016). For homogeneous cargo, the mixed-orientation heuristic recovered no more than 4.2% of the nominal load quantity, confirming that the binding decision lies upstream, at the packaging design stage, consistent with the system view of packaging logistics (Hellström & Saghir, 2007; Pålsson, 2018).

A second insight concerns design tightness. The designs with the highest nominal utilisation (Cases A and B, above 95%) were also the most fragile. Maximising nominal cube utilisation without a tolerance budget produces designs that sit on the edge of a threshold, so that most real cartons fall on the wrong side of it. Case D, with a lower nominal utilisation of 91.8% but positive margins, achieved the best expected outcome in the Monte Carlo simulation. Robust design therefore requires a trade-off between nominal utilisation and the probability of achieving it, a principle well established in tolerance design for mechanical assemblies (Chase & Parkinson, 1991) but rarely applied to transport packaging.

Sources of error and responsibility in the packaging supply chain

The scenario analysis links each error source to an actor and a control point. Omission of board caliper is a specification error that arises at the interface between the product designer, the buyer and the corrugated converter, often because specifications do not state whether dimensions are inside, outside or manufacturing dimensions. It was the largest single source of loss and is entirely preventable through unambiguous specifications that state the dimension type and flute profile, as recommended by box style codes (FEFCO & ESBO, 2007). Converting tolerance is a process capability issue for the converter and is controllable through statistical process control (Montgomery, 2019); its effect grows with the number of cartons per row and was decisive only for zero-slack designs. Panel bulge depends on board grade, fill level, product rigidity and humidity, and therefore links container utilisation to structural design and box compression performance (McKee et al., 1963; Frank, 2014). Rounding into imperial units is a communication error between buyer and supplier that can add up to 12.7 mm per dimension and whose effect is essentially random from the designer's perspective, as illustrated by the contrast between Case C (2.3% loss) and Case D (25.0% loss).

The non-additivity of errors has practical implications for diagnosis. Because only the error that first exhausts the slack on a given axis matters, correcting one error source may produce no improvement if another source on the same axis is still active. Root-cause analysis of container void space should therefore be conducted axis by axis, comparing the accumulated dimension of each row with the usable container dimension, rather than by evaluating error sources in isolation.

Managerial implications

Four implications follow for packaging engineers and logistics managers. First, carton specifications should state outer dimensions explicitly, together with inside dimensions, board grade, flute profile and caliper, and should be issued in a single unit system. Second, the target loading pattern should be fixed first and the permissible outer dimensions derived from Equation (5), so that the carton is designed for the container rather than the container being filled with whatever carton results. Third, a tolerance budget should be allocated among converting tolerance, bulge and clearance, and verified through first-article inspection of filled cartons rather than empty blanks. Fourth, a trial load or digital load plan using measured filled-carton dimensions should be part of the packaging approval process. These measures cost little compared with the additional freight of USD 28,800 to USD 277,200 per SKU per year indicated in Table 6, and they also reduce the need for dunnage and the risk of transit damage associated with loose stowage.

For Vietnamese exporters, many of which produce to buyer specifications issued in inches and source cartons from multiple local converters, the findings suggest that harmonised specification practice and converter capability are economically significant elements of logistics performance. Training in packaging logistics for engineers and logistics staff could therefore be a cost-effective measure for improving export competitiveness.

Limitations

The study has several limitations. First, the cases are stylised designs rather than data from specific firms, and inside container dimensions vary between container manufacturers; empirical validation with measured cartons and actual load reports is needed. Second, the model considers homogeneous, floor-loaded, volume-limited cargo; palletised loads, mixed SKUs and weight-limited cargo involve additional constraints. Third, the error distributions, in particular the bulge allowance and the systematic error in E3, were assumed rather than estimated from production data, and bulge was treated as uniform although it varies with stack position and humidity. Fourth, the loading heuristic does not exploit more complex non-guillotine patterns, which could recover slightly more space. Fifth, freight rates were illustrative and the analysis did not include warehousing, handling or emissions costs, which would increase the economic impact.

CONCLUSION

This study examined how dimensional errors in corrugated shipping carton design propagate into container void space. By combining a tolerance stack-up model with a load-planning heuristic, it showed that load quantity responds to dimensional error as a step function, so that errors of a few millimetres can remove 13% to 30% of cartons from a container. Omitting the board caliper reduced load quantity by 25.2% to 38.7%, and the combined effect of converting tolerance, panel bulge and imperial rounding reduced it by 15.8% to 32.0%, raising the void ratio to 27.2–37.6%. Load-planning optimisation recovered only a small part of these losses, indicating that container void space for homogeneous cargo is primarily a design-stage defect rather than a loading-stage problem.

The principal contributions are an explanation of the discontinuous mechanism linking carton dimensions to container utilisation, a decomposition of losses into error sources assignable to specific actors, and a permissible-dimension rule that converts a target loading pattern into a verifiable

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carton specification. Future research should validate the model with industrial data from carton converters and exporters, estimate empirical distributions of converting tolerance and bulge for common board grades, extend the analysis to palletised and mixed loads, and integrate the dimensional model with structural design so that board grade, compression strength and container utilisation can be optimised jointly. From a policy perspective, industry associations and trade promotion agencies could support standardised metric specification templates and training in packaging logistics for small and medium-sized exporters.

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ETHICAL AND COMPLIANCE STATEMENTS

Plagiarism statement: The author confirms that this manuscript is original and that the similarity index will not exceed 15%. **Conflict of interest:** The author declares no conflict of interest. **Funding:** This research received no external funding. **Research ethics:** The study is based on computational modelling and did not involve human participants or animals; ethics approval was therefore not required. **Data availability:** The simulation code and parameters are available from the author on reasonable request.

REFERENCES

1) Azzi, A., Battini, D., Persona, A., & Sgarbossa, F. (2012). Packaging design: General framework and research agenda. Packaging Technology and Science, 25(8), 435–456. https://doi.org/10.1002/pts.993
2) Bischoff, E. E., & Ratcliff, M. S. W. (1995). Issues in the development of approaches to container loading. Omega, 23(4), 377–390. https://doi.org/10.1016/0305-0483(95)00015-G
3) Bortfeldt, A., & Gehring, H. (2001). A hybrid genetic algorithm for the container loading problem. European Journal of Operational Research, 131(1), 143–161. https://doi.org/10.1016/S0377-2217(00)00055-2
4) Bortfeldt, A., & Wäscher, G. (2013). Constraints in container loading – A state-of-the-art review. European Journal of Operational Research, 229(1), 1–20. https://doi.org/10.1016/j.ejor.2012.12.006
5) Chase, K. W., & Parkinson, A. R. (1991). A survey of research in the application of tolerance analysis to the design of mechanical assemblies. Research in Engineering Design, 3(1), 23–37. https://doi.org/10.1007/BF01580066
6) Dowsland, K. A., & Dowsland, W. B. (1992). Packing problems. European Journal of Operational Research, 56(1), 2–14. https://doi.org/10.1016/0377-2217(92)90288-K
7) Dyckhoff, H. (1990). A typology of cutting and packing problems. European Journal of Operational Research, 44(2), 145–159. https://doi.org/10.1016/0377-2217(90)90350-K
8) FEFCO & ESBO. (2007). International fibreboard case code (11th ed.). European Federation of Corrugated Board Manufacturers.
9) Frank, B. (2014). Corrugated box compression – A literature survey. Packaging Technology and Science, 27(2), 105–128. https://doi.org/10.1002/pts.2019
10) García-Arca, J., Prado-Prado, J. C., & González-Portela Garrido, A. T. (2014). "Packaging logistics": Promoting sustainable efficiency in supply chains. International Journal of Physical Distribution & Logistics Management, 44(4), 325–346.
11) George, J. A., & Robinson, D. F. (1980). A heuristic for packing boxes into a container. Computers & Operations Research, 7(3), 147–156. https://doi.org/10.1016/0305-0548(80)90001-5
12) Hellström, D., & Saghir, M. (2007). Packaging and logistics interactions in retail supply chains. Packaging Technology and Science, 20(3), 197–216. https://doi.org/10.1002/pts.754
13) International Organization for Standardization. (2012). ISO 3394:2012 Packaging – Complete, filled transport packages and unit loads – Dimensions of rigid rectangular packages. ISO.
14) International Organization for Standardization. (2020). ISO 668:2020 Series 1 freight containers – Classification, dimensions and ratings. ISO.
15) Levinson, M. (2016). The box: How the shipping container made the world smaller and the world economy bigger (2nd ed.). Princeton University Press.
16) McKee, R. C., Gander, J. W., & Wachuta, J. R. (1963). Compression strength formula for corrugated boxes. Paperboard Packaging, 48(8), 149–159.
17) Montgomery, D. C. (2019). Introduction to statistical quality control (8th ed.). Wiley.
18) Pålsson, H. (2018). Packaging logistics: Understanding and managing the economic and environmental impacts of packaging in supply chains. Kogan Page.
19) Ramos, A. G., Oliveira, J. F., Gonçalves, J. F., & Lopes, M. P. (2016). A container loading algorithm with static mechanical equilibrium stability constraints. Transportation Research Part B: Methodological, 91, 565–581. https://doi.org/10.1016/j.trb.2016.06.003
20) Twede, D., Selke, S. E. M., Kamdem, D.-P., & Shires, D. (2015). Cartons, crates and corrugated board: Handbook of paper and wood packaging technology (2nd ed.). DEStech Publications.
21) UNCTAD. (2024). Review of maritime transport 2024: Navigating maritime chokepoints. United Nations Conference on Trade and Development.
22) Wäscher, G., Haußner, H., & Schumann, H. (2007). An improved typology of cutting and packing problems. European Journal of Operational Research, 183(3), 1109–1130. https://doi.org/10.1016/j.ejor.2005.12.047
23) Zhao, X., Bennell, J. A., Bektaş, T., & Dowsland, K. (2016). A comparative review of 3D container loading algorithms. International Transactions in Operational Research, 23(1–2), 287–320. https://doi.org/10.1111/itor.12094

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